Formulaire en coordonnées cylindriques {r, θ, z}

Les composantes du vecteur vitesse en coordonnées cylindriques sont $\vec{v}\{u, v, w\}$

Gradient

$$\overrightarrow{\nabla}\mathrm{f} = \frac{\partial f}{\partial r} + \frac{1}{r}\frac{\partial f}{\partial \theta} + \frac{\partial f}{\partial z}$$

Dérivée particulaire

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial r} + \frac{v}{r}\frac{\partial f}{\partial \theta} + w\frac{\partial f}{\partial z}$$

Laplacien

$$\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

Tenseur gradient de vitesse

$$\overline{\overline{\nabla \vec{v}}} = \begin{bmatrix} \frac{\partial u}{\partial r} & \frac{1}{r}\left(\frac{\partial u}{\partial \theta} - v\right) & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial r} & \frac{1}{r}\left(\frac{\partial v}{\partial \theta} + u\right) & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial r} & \frac{1}{r}\frac{\partial w}{\partial \theta} & \frac{\partial w}{\partial z} \end{bmatrix}$$

Tenseur vitesse de déformation

$$\overline{\overline{\mathrm{D}}} = \begin{bmatrix} \frac{\partial u}{\partial r} & \frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}\right) & \frac{1}{r}\left(\frac{\partial v}{\partial \theta} + \frac{u}{r}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right) & \frac{\partial w}{\partial z} \end{bmatrix}$$

Equation de la dynamique

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + f_r - \rho\gamma_r = 0$$
$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} + f_\theta - \rho\gamma_\theta = 0$$
$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} + f_z - \rho\gamma_z = 0$$

Equation de Navier-Stokes incompressible en absence de force de volume

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,u)\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w\frac{\partial v}{\partial z} + \frac{u\,v}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \nu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v)\right) + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u}{\partial \theta} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^2 w}{\partial z^2}\right)$$